Turns around a point

In the absence of wind one can fly a circular course at a constant bank angle. The bank angle required is easily computed:

With a bank angle ψ, vertical balance tells us

$$L\cos(\psi) = Mg$$

where L is the lift, and M the mass of the airplane. Equating the cenrtifugal force to the horizontal component of lift gives:

$$L\sin(\psi) = Mv^2 / R$$

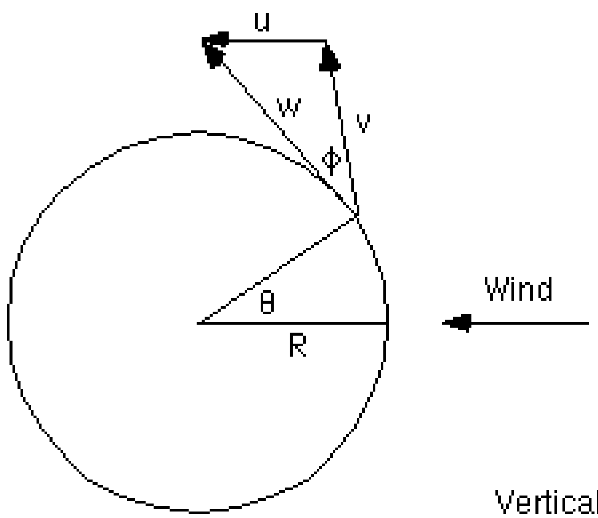
where v is the TAS and R the radius of turn. Eliminating the lift gives:

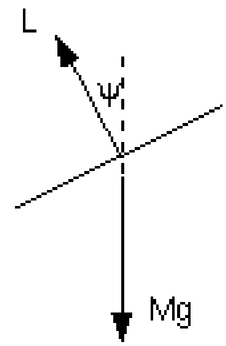
$$\tan(\psi) = v^2 / Rg$$

$$\psi = \arctan(v^2 / Rg)$$

The question arises as to what happens iwhen one wants to track a circle over the ground, as in the Private Pilot "Turns around a point", when there is a wind There are two effects, first, the wind affects the groundspeed so that a steeper bank is required downwind than upwind. The second effect is that a crab is required during parts of the maneuver. Because the horizontal component of lift is no longer directed towards the center of the turn, a greater bank angle is required for the groundspeed. The component of the lift along the ground track is resonsible for the acceleration and deceleration.

The figure below illustrates the situation.





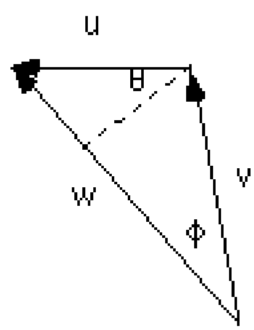
Vertical balance: L cos(ψ) = Mg

Turns around a point.

Horizontal component of lift =Lsin(ψ) =Mg tan(ψ)

Centrifugal Force= Component of horizontal component of lift towards center

$Mw^2/R = Mg\tan(\psi)\cos(\phi)$



u cos(θ) =v sin (ϕ)

Wind triangle

Now the balance between centrifugal force and the component of lift directed to the center becomes:

$$Mw^2 / R = Mg\tan(\psi)\cos(\phi)$$

From the wind triangle, we have:

$$u\cos(\theta) = v\sin(\phi)$$

$$w = u\sin(\theta) + v\cos(\phi) = u\sin(\theta) + v\sqrt{1-(u\cos(\theta)/\ v)^2}$$

Thus the bank angle is given by:

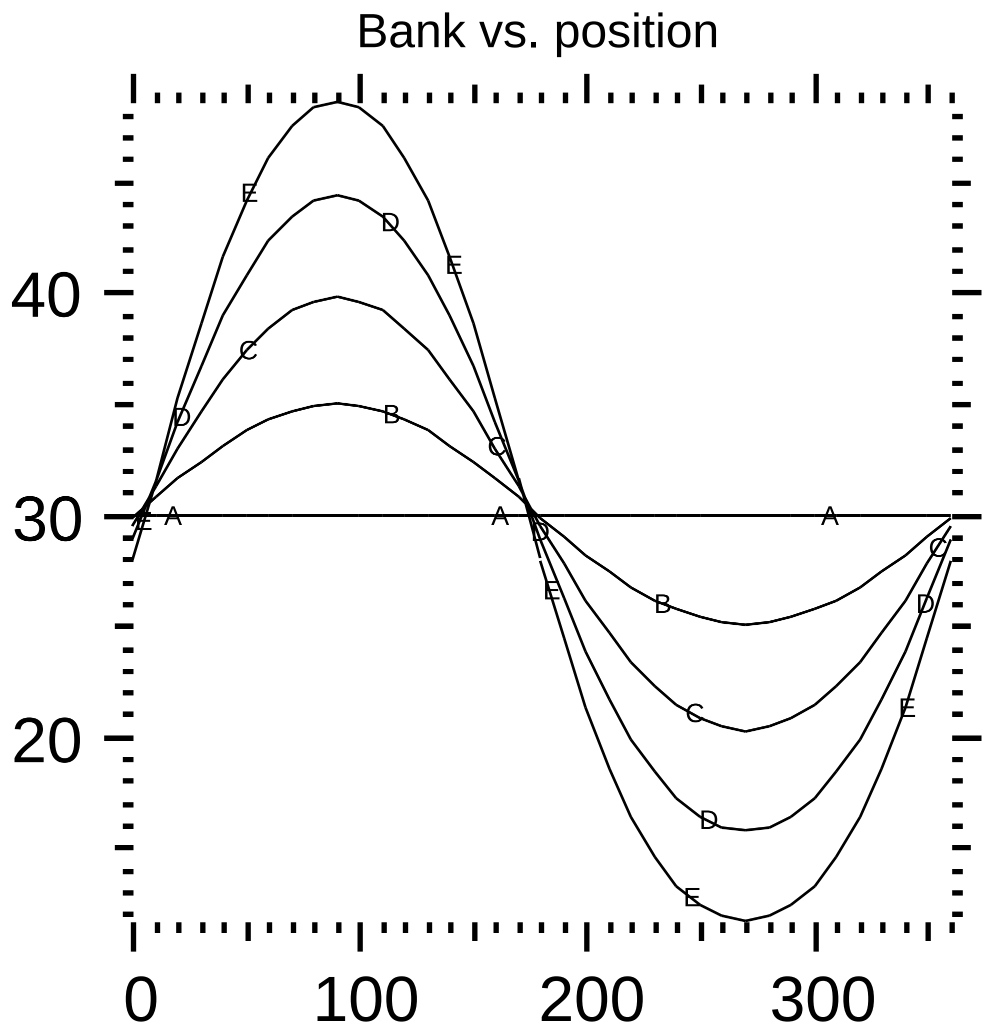
$$\psi = \arctan\left\{ \frac{v^2\left(u\sin(\theta)/v + \sqrt{1-(u\cos(\theta)/v)^2}\right)^2}{Rg\sqrt{1-(u\cos(\theta)/v)^2}} \right\}$$

Note first that reduces to the no wind result above when u=0. Downwind, when $\theta=\pi/2$, we obtain

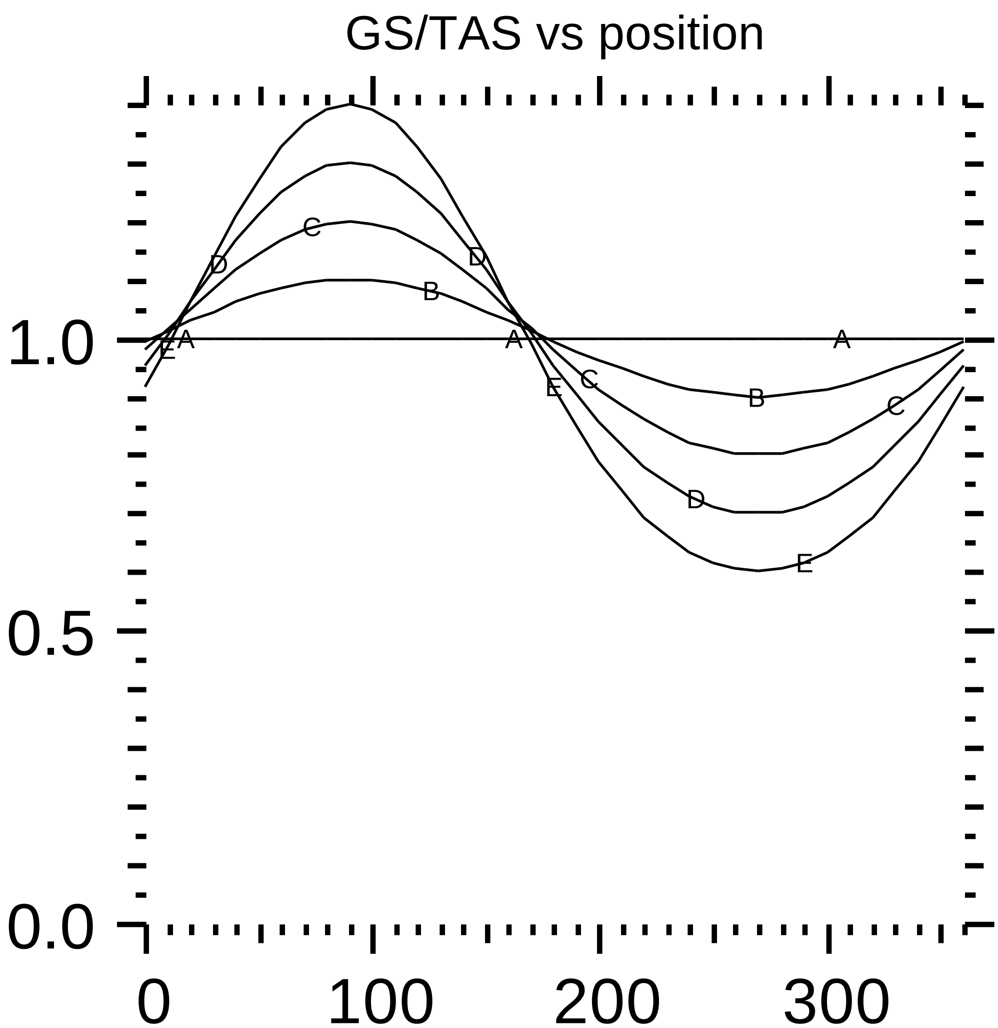
$$\psi = \arctan((v+u)^2 / Rg)$$

which is the expected bank for a groundspeed of v+u with no crab.

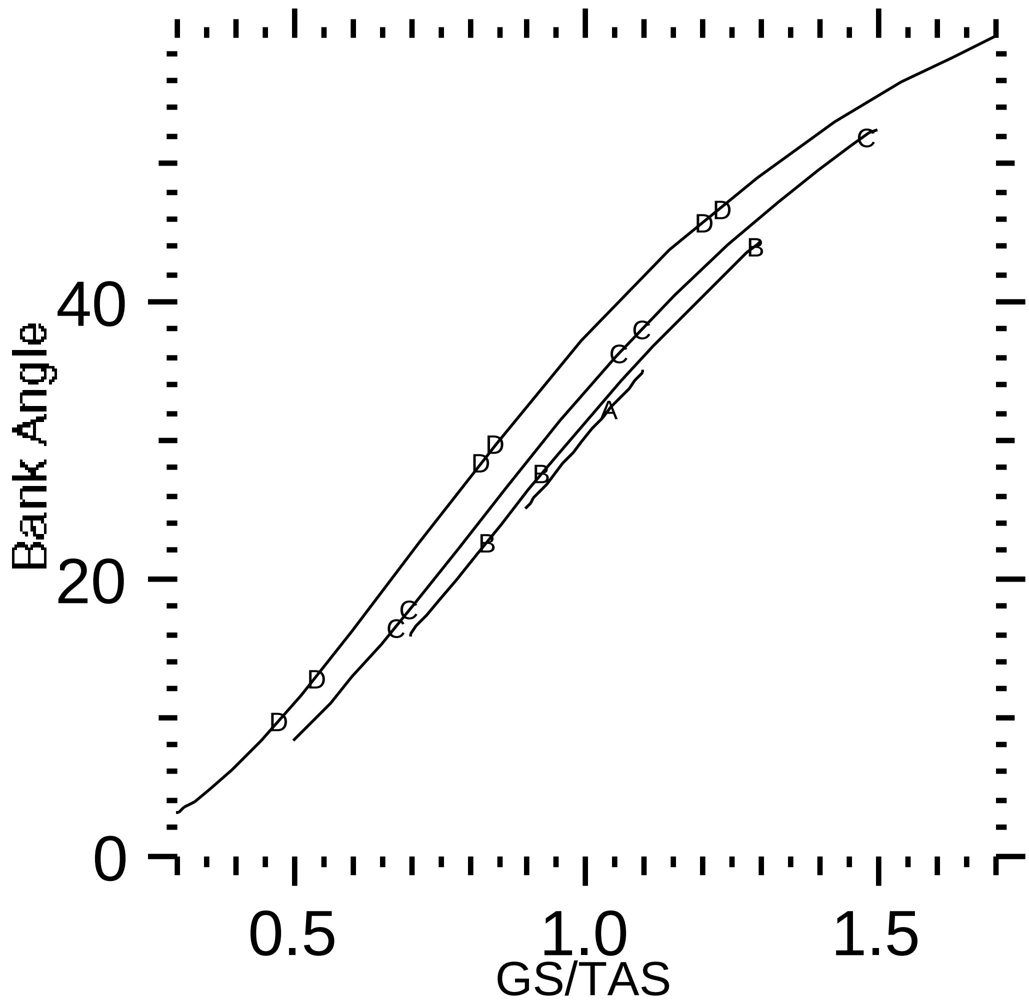
We'll fix TAS and turn radius and look at the variation of bank around the turn as the wind strength varies.



Here we look at a turn which requires a 30 degree bank in no wind. Curves A,B,C,D and E are with no wind and with winds of 10%, 20%, 30% and 40% of the TAS respectively. At the 90 and 270 points the plane is directly downwind and upwind respectively. At 0 and 180, the plane is crosswind, and as the wind speed increases, the groundspeed decreases because of the required crab.



We now show bank angle vs. gs/TAS for a turn that requires a 30 degree band in no wind. Curves A, B, C and D are for winds of 10%, 30%, 50% and 70% of the TAS. We see that the required bank angle is primarily determined by the ground speed. However, there is the effect, decribed earlier, that as the wing gets stronger the crab angle increases, so that a smaller fraction of the horizontal component of lift is directed towards the center of the turn.


Bank Angle
40
20
0
0.5
1.0
1.5
GS/TAS
A
B
C
D